Calculus Booklet of Formulas

GREEK ALPHABET

Capital	Small	Name
Α	α	Alpha
Β	β	Beta
Γ	γ	Gamma
Δ	δ	Delta
Ε	ε	Epsilon
Ζ	ζ	Zeta
Η	η	Eta
Θ	θ	Theta
Ι	ι	Iota
Κ	κ	Kappa
Λ	λ	Lambda
Μ	μ	Mu
Ν	ν	Nu
Ξ	ξ	Xi
Ο	ο	Omicron
Π	π	Pi
Ρ	ρ	Rho
Σ	σ	Sigma
Τ	τ	Tau
Υ	υ	Upsilon
Φ	φ	Phi
Χ	χ	Chi
Ψ	ψ	Psi
Ω	ω	Omega

ABSOLUTE VALUE

$$|x| = \begin{cases} x & for\ x \geq 0 \\ -x & for\ x < 0 \end{cases}$$

$$|x| = \sqrt{x^2}$$

ARITHMETIC SEQUENCES

The n th term of an arithmetic sequence with first term a_1 and common difference d is given by

$$a_n = a_1 + (n-1)d$$

The sum S_n of the first n terms of an arithmetic sequence is given by

$$S_n = \frac{n}{2}(a_1 + a_n)$$

GEOMETRIC SEQUENCES

The n th term of a geometric sequence with first term a_1 and common ratio r is given by

$$a_n = a_1 r^{n-1}$$

The sum S_n of the first n terms of a geometric sequence is given by

$$S_n = \frac{a_1(1-r^n)}{(1-r)}$$

The sum S of an infinite geometric sequence with $|r| < 1$ is given by

$$S = \frac{a_1}{(1-r)}$$

EXPONENTAILS AND LOGARITHMS

$y = \log_b(x)$ if and only if $b^y = x$

$\log_b(xy) = \log_b(x) + \log_b(y)$

$\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$

$\log_b x^r = r\log_b(x)$

$b^{\log_b(x)} = x$

$\log_b(b^x) = x$

$\log_b(1) = 0$

$\log_b(b) = 1$

$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$, change of base

BINOMIAL THEOREM

n positive integer

$$(x+y)^n = x^n + {}_nC_1 x^{n-1} y + {}_nC_2 x^{n-2} y^2 + \cdots$$
$$+ {}_nC_r x^{n-r} y^r + \ldots + y^n$$

Where ${}_nC_r = \frac{n!}{r!(n-r)!}$

EXPONENTS AND RADICALS

$x^0 = 1$

$x^{-r} = \frac{1}{x^r} = (\frac{1}{x})^r$

$\frac{1}{x^{-r}} = x^r$

$x^r x^s = x^{r+s}$

$(x^r)^s = x^{rs}$

$(\frac{x}{y})^r = \frac{x^r}{y^r}$

$\frac{x^r}{y^s} = y^{r-s}$

$(xy)^r = x^r y^r$

$(\frac{x}{y})^{-r} = (\frac{y}{x})^r$

$x^{1/n} = \sqrt[n]{x}$

$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

$x^{n/m} = (\sqrt[m]{x})^n$

$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

DERIVATIVES

k is a constant.

$f(x)$	$f'(x)$
k	0
$k\ f(x)$	$k\ f'(x)$
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\sin^{-1} x$, $\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$, $\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$, $\arctan x$	$\frac{1}{1+x^2}$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\sec h^2 x$
$\coth x$	$-\csc h^2 x$
$\sec hx$	$-\sec hx \tanh x$
$\csc hx$	$-\csc hx \coth x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{x^2+1}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\coth^{-1} x$	$-\frac{1}{1-x^2}$

INDEFINITE INTEGRALS

A constant of integration should be added in all cases.

$f(x)$	$\int f(x)dx$
k	kx
$k\ f(x)$	$k\int f(x)dx$
$\frac{1}{x}$	$\ln\lvert x\rvert$
e^x	e^x
$\ln x$	$x\ln x - x$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln(\cos x)$
$\cot x$	$\ln(\sin x)$
$\sec x$	$\ln(\tan(\frac{x}{2}+\frac{\pi}{4}))$
$\csc x$	$\ln(\tan(\frac{x}{2}))$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln(\cosh x))$
$\coth x$	$\ln(\lvert\sinh x\rvert))$
$\sec hx$	$2\tan^{-1}(e^x)$
$\csc hx$	$\ln(\lvert\tanh(\frac{x}{2})\rvert)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}(\frac{x}{a})$
$\frac{1}{\sqrt{a^2+x^2}}$	$\sinh^{-1}(\frac{x}{a})$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}(\frac{x}{a})$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}(\frac{x}{a})$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln(\frac{x-a}{x+a})$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln(\frac{a+x}{a-x})$

TAYLOR SERIES (about $x=a$)

$$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f''(a)$$
$$+\cdots+\frac{(x-a)^n}{n!}f^{(n)}(a)+\cdots$$

MACLAURIN SERIES

$$f(x)=f(0)+xf'(0)+\frac{x^2}{2!}f''(0)$$
$$+\cdots+\frac{x^n}{n!}f^{(n)}(0)+\cdots$$

REAL FOURIER SERIES (Period T)

$$f(t)=\frac{1}{2}a_0+\sum_{m=1}^{\infty}a_m\cos(\frac{2\pi m}{T}t)+\sum_{m=1}^{\infty}b_m\sin(\frac{2\pi m}{T}t)$$

Where

$$a_m=\frac{2}{T}\int_0^T f(t)\cos(\frac{2\pi m}{T}t)dt$$

$$b_m=\frac{2}{T}\int_0^T f(t)\sin(\frac{2\pi m}{T}t)dt$$

COMPLEX FOURIER SERIES (Period T)

$$f(t)=\sum_{m=-\infty}^{\infty}c_m\exp(i\frac{2\pi m}{T}t)$$

where

$$c_m=\frac{1}{T}\int_0^T f(t)\exp(-i\frac{2\pi m}{T}t)dt$$

Relationship between the above coefficients of the real and complex forms.

$$c_m=\frac{1}{2}(a_m-ib_m)) \quad m>0$$

$$c_0=\frac{1}{2}a_0$$

$$c_m=\frac{1}{2}(a_{-m}+ib_{-m}) \quad m<0$$

FOURIER TRANSFORM PAIR

$$\tilde{F}(\omega)=\int_{-\infty}^{\infty}f(t)\exp(-i\omega t)dt$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{F}(\omega)\exp(i\omega t)dt$$

POWER SERIES FOR SOME FUNCTIONS

$$e^x=1+x+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}+\cdots \qquad \text{for all } x$$

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}\cdots+(-1)^{n+1}\frac{x^n}{n!}+\cdots$$

$$-1<x\le 1$$

$$\sin x=\frac{e^{jx}-e^{-jx}}{2}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$$

$$\sin^{-1}x=\arcsin x=x+\frac{1}{2}\frac{x^3}{3}+\frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7}+\cdots$$

$$\cos x=\frac{e^{jx}+e^{-jx}}{2}=x-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$

$$\tan x=x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7+\cdots$$

$$\tan^{-1}x=\arctan x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\cdots$$

$$\sinh x=\frac{e^x-e^{-x}}{2}=x+\frac{x^3}{3!}+\frac{x^5}{5!}+\frac{x^7}{7!}+\cdots$$

$$\sinh^{-1}x=\arcsin h\,x=x-\frac{1}{2}\frac{x^3}{3}+\frac{1\cdot 3}{2\cdot 4}\frac{x^5}{5}-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\frac{x^7}{7}+\cdots$$

$$\cosh x=\frac{e^x+e^{-x}}{2}=x+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\tanh x=x-\frac{1}{3}x^3+\frac{2}{15}x^5-\frac{17}{315}x^7+\cdots$$

$$\tanh^{-1}x=\arctan hx=x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+\cdots$$

INTEGER SERIES

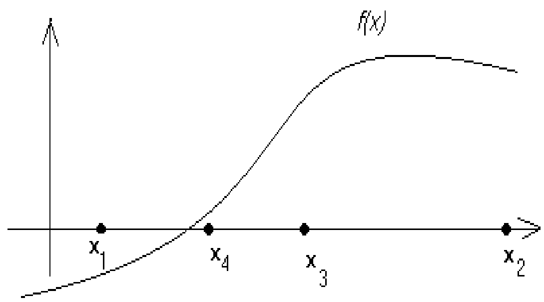
$$\sum_{i=1}^{N} i = 1+2+3+\cdots+N = \frac{1}{2}N(N+1)$$

$$\sum_{i=1}^{N} i^2 = 1^2+2^2+3^2+\cdots+N^2 = \frac{1}{6}N(N+1)(2N+1)$$

$$\sum_{i=1}^{N} i^3 = 1^3+2^3+3^3+\cdots+N^3 = [\frac{1}{2}N(N+1)]^2$$

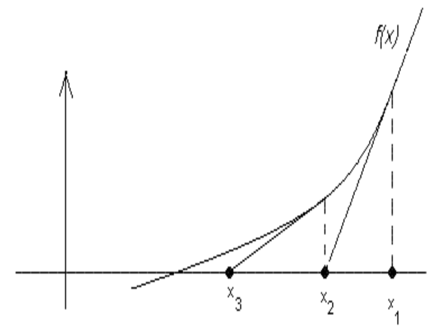
NUMERICAL SOLUTIONS OF f(x) = 0

BISECTION METHOD



x_1 and x_2 are estimates of the root that are on opposite sides of the root (i.e. such that $f(x_1)\cdot f(x_2) < 0$). The next estimate is given by the arithmetic mean of the previous two estimates, $x_3 = \frac{x_1 + x_2}{2}$. Discard whichever of x_1 or x_2 is on the same side of the root as x_3 (x_2 in the diagram) and repeat the process. The root of $f(x) = 0$ always lies between the last two estimates. The bisection method is slow but always converges for a 'well behaved' function.

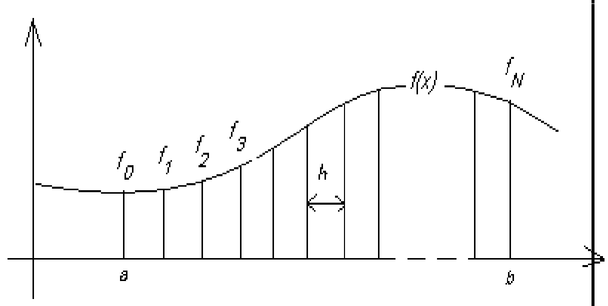
NEWTON METHOD



Start with a single estimate x_1 of the root; then successive estimates are given in terms of the previous one by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

NUMERICAL INTEGRATION



Let $h = \dfrac{b-a}{N}$

Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}[f_0 + 2f_1 + 2f_2 + 2f_3 + 2f_4 + \cdots + 2f_{N-2} + 2f_{N-1} + f_N]$$

Simpson rule

The number of intervals N must be even

$$\int_a^b f(x)dx \approx \frac{h}{3}[f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + \cdots + 2f_{N-2} + 4f_{N-1} + f_N]$$

LAPLACE TRANSFORMS

$$L(f(t)) = F(s) = \int_{0^-}^{\infty} f(t)\exp(-st)dt$$

$f(t)$	$F(s)$	comment
1	$\frac{1}{s}$	
t^n	$\frac{n!}{s^{n+1}}$	
e^{-at}	$\frac{1}{s+a}$	
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$	comments
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$	
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^2}$	
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$	
$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$	
$\cosh \omega t$	$\frac{s}{s^2-\omega^2}$	
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$	
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$	
$\delta(t-\tau)$	$e^{-s\tau}$	dirac delta function
$H(t-\tau)$	$\frac{1}{s}e^{-s\tau}$	step function,
$e^{-at}f(t)$	$F(s+a)$	damping in t becomes
$f(kt)$	$\frac{1}{k}F(\frac{s}{k})$	scale change
$tf(t)$	$-\frac{dF}{ds}$	first Derivative
$f'(t)$	$sF(s)-f(0)$	first derivative
$f'''(t)$	$s^2F(s)-sf(0)-f'(0)$	second derivative
$\int_0^t f(x)dx$	$\frac{1}{s}F(s)$	Integral of f(t) w.r.t t
$\int_0^t f_1(x)f_2(t$	$F_1(s)F_2(s)$	convolution integral
$af(t)+bg($	$aF(s)+bG(s)$	Linearity