Calculus Worksheet: Differentiation of Functions (1)

Use differentiation rules to find *dy /dx* for each function given below

1. $y = [e^{2x^2+1}]^3$

let $u = e^{2x^2+1} \Rightarrow y = u^3$

chain rule: $\frac{dy}{dx} = 3 \cdot u^{3-1} \cdot \frac{du}{dx}$

$\frac{du}{dx} = 4x \cdot e^{2x^2+1} \Rightarrow \frac{dy}{dx} = 12x\left[e^{2x^2+1}\right]^3$

2. $y = \sqrt[5]{\frac{x^2+1}{1-x^3}}$

let $u = \frac{x^2+1}{1-x^3} \Rightarrow y = u^{\frac{1}{5}}$

chain rule: $\frac{dy}{dx} = \frac{1}{5} \cdot u^{\frac{1}{5}-1} \cdot \frac{du}{dx}$

$\frac{du}{dx} = \frac{2x(1-x^3)+3x^2(x^2+1)}{(1-x^3)^2} = \frac{x^4+3x^2+2x}{(1-x^3)^2}$

$\frac{dy}{dx} = \frac{1}{5} \cdot \left(\frac{x^4+3x^2+2x}{(1-x^3)^2}\right) \cdot \left(\frac{x^2+1}{1-x^3}\right)^{-\frac{4}{5}}$

$= \frac{1}{5} \frac{(x^4+3x^2+2x)}{(1-x^3)^{6/5}(x^2+1)^{4/5}}$

3. $y = \arcsin(\frac{x^2}{x^2+1})$

let $u = \frac{x^2}{x^2+1} \Rightarrow y = \arcsin(u)$.

Chain rule : $\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$.

$$\frac{du}{dx} = \frac{2x(x^2+1) - 2x(x^2)}{(x^2+1)} = \frac{2x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{2x}{(x^2+1)^2} \cdot \frac{1}{\sqrt{1 - \frac{x^4}{(x^2+1)^2}}} = \frac{2x}{(x^2+1)\sqrt{2x^2+1}}$$

4. $y = x^{\sin x}$, $x > 0$

Two methods.

method (1)

$\ln y = \sin x \ln x$.

$$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \ln x + \sin x \frac{1}{x}$$

$$\frac{dy}{dx} = y\left[\cos x \ln x + \sin x \frac{1}{x}\right]$$

$$= \left[\cos x \ln x + \frac{\sin x}{x}\right] x^{\sin x}$$

method (2)

$y = x^{\sin x} = e^{\ln x \cdot \sin x}$.

$$\frac{dy}{dx} = \left[\frac{d}{dx}(\ln x \cdot \sin x)\right] e^{\ln x \sin x}$$

$$= \left(\frac{1}{x}\sin x + \ln x \cos x\right) e^{\ln x \sin x}$$

$$= \left(\ln x \cos x + \frac{\sin x}{x}\right) x^{\sin x}.$$

5. $y = \dfrac{\ln(x+1)}{\ln(x-3)}$

let $u = \ln(x+1)$ and $V = \ln(x-3)$

$$\frac{dy}{dx} = \frac{u'V - uV'}{V^2} = \frac{\frac{1}{x+1} \cdot \ln(x-3) - \ln(x+1)\frac{1}{x-3}}{\ln(x-3)^2}$$

$$= \frac{(x-3)\ln(x-3) - (x+1)\ln(x+1)}{(x+1)(x-3)\ln(x-3)^2}.$$

6. $y = \ln(\ln(x))$

let $u = \ln(x)$ $\Rightarrow y = \ln u.$

chain rule : $\dfrac{dy}{dx} = \dfrac{du}{dx} \cdot \dfrac{1}{u}.$

$$= \frac{1}{x} \cdot \frac{1}{\ln x}.$$

$$= \frac{1}{x \ln x}.$$

7. $y = \arctan(\frac{x+2}{x-3})$

let $u = \frac{x+2}{x-3} \Rightarrow y = \arctan(u)$.

Chain rule : $\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{1}{u^2+1}$

$\frac{dy}{dx} = \frac{-5}{(x-3)^2} \cdot \frac{1}{\left(\frac{x+2}{x-3}\right)^2+1} = \frac{-5}{2x^2-2x+13}$

8. $y = \sin(\cos(x+2))$

let $u = \cos(x+2) \Rightarrow y = \sin u$.

Chain rule: $\frac{dy}{dx} = \frac{du}{dx} \cdot \cos u$.

$\Rightarrow \frac{dy}{dx} = -\sin(x+2) \cdot \cos(\cos(x+2))$