Calculus Worksheet: Differentiation of Functions (2)

Find the first derivative of the functions

1. $y=(4x-4)^4$

let $u = 4x-4 \Rightarrow \frac{du}{dx} = 4$.

$y = u^4$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 4u^3 \cdot 4 = \underline{16(4x-4)^3}$

2. $y=\cos(\sin(x)+3x)$

let $u = \sin(x)+3x$, $\frac{du}{dx} = \cos(x)+3$

$y = \cos(u)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\sin(u)\cdot(\cos(x)+3)$

$= \underline{-\sin(\sin(x)+3x)(\cos(x)+3)}$

3. $y=e^{x^2+4x}$

let $u = x^2+4x$, $\frac{du}{dx} = 2x+4$

$y = e^u$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = e^u \cdot (2x+4)$

$= \underline{e^{x^2+4x}\cdot(2x+4)}$

4. $y=\sin^{-1}(\sqrt{x}-1)$, $0 \le x \le 4$

let $u = \sqrt{x}-1$, $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$y = \sin^{-1}(u)$, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{\sqrt{1-(\sqrt{x}-1)^2}} \cdot \frac{1}{2\sqrt{x}}$

5. $y = \dfrac{2}{\sqrt{(1-x^3)^5}}$

let $u = 1-x^3$, $\dfrac{du}{dx} = -3x^2$

$y = \dfrac{2}{\sqrt{u^5}} = 2 \cdot u^{-5/2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = 2(-5/2)\, u^{-5/2-1}\, (-3x^2)$

$= \dfrac{15x^2}{\sqrt{(1-x^3)^7}}$

6. $y = \ln(2x^2+4)^2$

let $u = (2x^2+4)^2$, $\dfrac{du}{dx} = 2(4x)(2x^2+4)$.

$y = \ln(u)$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = \dfrac{1}{u} \cdot 8x(2x^2+4) = \dfrac{4x}{x^2+2}$

7. $y = \ln^2(2x^2+4)$

let $u = 2x^2+4$, $\dfrac{du}{dx} = 4x$

$y = \ln^2(u)$.

let $v = \ln u \Rightarrow y = v^2$

$\dfrac{dy}{dx} = \dfrac{dy}{dv} \cdot \dfrac{dv}{du} \cdot \dfrac{du}{dx} = 2v \cdot \left(\dfrac{1}{u}\right) \cdot 4x$

$= 2\ln(2x^2+4) \cdot \dfrac{4x}{2x^2+4}$

$= 2\ln(2x^2+4)\dfrac{2x}{x^2+2}$

8. $y = 3u^{\frac{1}{2}} - 2u^{\frac{1}{3}}$ with $u = 2x^3 + x$, $\dfrac{du}{dx} = 6x^2 + 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(3 \cdot \frac{1}{2} u^{\frac{1}{2}-1} - 2\left(\frac{1}{3}\right) u^{\frac{1}{3}-1}\right)(6x^2+1)$$

$$= \left[\frac{3}{2} \frac{1}{\sqrt{2x^3+x}} - \frac{2}{3} \frac{1}{\sqrt[3]{(2x^3+x)^2}}\right](6x^2+1)$$

9. $y = \sqrt[3]{(2x^2+x-1)^2 - 2}$

let $u = (2x^2+x-1)^2 - 2$, $\dfrac{du}{dx} = 2(4x+1)(2x^2+x-1)$

$$y = \sqrt[3]{u} = u^{1/3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{1}{3} u^{1/3-1}\right)(2)(4x+1)(2x^2+x-1)$$

$$= \frac{2}{3} \cdot \frac{1}{\sqrt[3]{\left((2x^2+x-1)^2-2\right)^2}} \cdot (4x+1)(2x^2+x-1)$$