## Calculus Worksheet: Questions on Differentiation of Inverse Functions (2)

Question 1:

If $f(0) = -3$ and $f'(0) = 6$, find $(f^{-1})'(-3)$.

$$f(0) = -3 \iff f^{-1}(-3) = 0$$

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(0)} = \frac{1}{6}.$$

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Question 2:

$f$ is a function given by

$$f(x) = \sin(x) + x \,, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

1. Show that $f$ is a one to one function
2. Find $(f^{-1})'(0)$

① $f'(x) = \cos(x) + 1$ , for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ $\quad 0 \le \cos(x) \le 1$

$\Rightarrow \quad \cos(x) + 1 > 0 \Rightarrow f'(x) > 0$

$\Rightarrow f(x)$ is increasing for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\Rightarrow f$ is a one to one function.

② $(f^{-1})'(0) = \dfrac{1}{f'(f^{-1}(0))}$



Since $f(0) = 0 \iff f^{-1}(0) = 0$

$$= \frac{1}{f'(0)} = \frac{1}{2}.$$

Question 3:

Let $(f^{-1})'(x) = 2x$, $x \geq 0$ and $f^{-1}(2) = 3$, find $f'(x)$.

$$f^{-1}(x) = \int (f^{-1})'(x) \cdot dx = \int 2x \cdot dx = x^2 + k.$$

$$f^{-1}(2) = (2)^2 + k = 3 \quad \text{given.}$$

$$\Rightarrow \quad k = 3 - 4 = -1.$$

$$f^{-1}(x) = x^2 - 1.$$

method (1) : we now find $f(x)$ by finding the inverse of $f^{-1}(x)$.

$$y = x^2 - 1, \quad x \geq 0$$

$$x = y^2 - 1, \quad y \geq 0$$

$$y = \pm\sqrt{x+1}, \quad y \geq 0$$

$$f(x) = y = \sqrt{x+1}. \quad \Rightarrow \quad f'(x) = \frac{1}{2\sqrt{x+1}}.$$

method (2) :

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(x^2-1)} = 2x$$

$$\Rightarrow \quad f'(x^2 - 1) = \frac{1}{2x}$$

let $w = x^2 - 1 \Rightarrow x^2 = w + 1 \Rightarrow x = \sqrt{w+1}$, $x \geq 0$.

$$\Rightarrow \quad f'(w) = \frac{1}{2\sqrt{w+1}}$$

More examples at

www.analyzemath.com/calculus/Differentiation/inverse_trigonometric.html