

Calculus Worksheet: Domain and Range (1)

1. Find the domain of the following functions

a) $f(x) = \frac{1}{x-2}$

$$x-2 \neq 0 \Rightarrow x \neq 2$$

$$\text{domain: } \underline{(-\infty, 2) \cup (2, +\infty)}$$

b) $g(x) = \frac{1}{x^4+2}$

$$x^4+2 > 0 \Rightarrow \text{domain: } \underline{(-\infty, +\infty)}$$

c) $h(x) = \ln(x^2-9)$

$$x^2-9 > 0$$

$$\text{domain: } \underline{(-\infty, -3) \cup (3, +\infty)}$$

d) $i(x) = \sqrt{2-\sqrt{x+2}}$

$$\text{Condition (1): } x+2 \geq 0 \Rightarrow x \geq -2$$

$$\text{Condition (2): } 2 - \sqrt{x+2} \geq 0$$

$$2 \geq \sqrt{x+2}$$

$$4 \geq x+2$$

$$\Rightarrow x \leq 2$$

$$\text{domain: } \underline{[-2, 2]}$$

$$e) j(x) = \frac{1}{x^3 - 2}$$

$$x^3 - 2 \neq 0 \Rightarrow x \neq \sqrt[3]{2}$$

$$\text{Domain: } (-\infty, \sqrt[3]{2}) \cup (\sqrt[3]{2}, +\infty)$$

$$f) k(x) = \frac{1}{2 - \sqrt{x^2 - 1}}$$

$$\text{Condition (1): } x^2 - 1 \geq 0 \Rightarrow x \in (-\infty, -1] \cup [1, +\infty)$$

$$\text{Condition (2): } 2 - \sqrt{x^2 - 1} \neq 0 \Rightarrow 4 \neq x^2 - 1 \\ x \neq \pm \sqrt{5}$$

Domain:

$$g) l(x) = \frac{\sqrt{x+8}}{(x-6)\sqrt{|x|-4}}$$

$$(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, -1) \cup (1, \sqrt{5}) \cup (\sqrt{5}, +\infty)$$

$$\text{Condition (1): } x+8 \geq 0 \quad x \geq -8$$

$$\text{Condition (2): } x-6 \neq 0 \quad x \neq 6$$

$$\text{Condition (3): } |x|-4 > 0 \Rightarrow x \in (-\infty, -4) \cup (4, +\infty)$$

$$\text{Domain: } \underline{[-8, -4) \cup (4, 6) \cup (6, +\infty)}$$

2. Find the range of the following functions

$$a) f(x) = |x+2| - 5$$

$$|x+2| \geq 0 \Rightarrow \underbrace{|x+2| - 5}_{f(x)} \geq -5$$

$$\text{range: } \underline{[-5, +\infty)}$$

$$b) g(x) = 2 + \sqrt{x+5}$$

$$\sqrt{x+5} \geq 0 \Rightarrow \underbrace{2 + \sqrt{x+5}}_{g(x)} \geq 2$$

$$\text{range: } \underline{[2, +\infty)}$$

c) $h(x) = x^2 + 4x + 8$

$= (x+2)^2 + 4$ complete square

$(x+2)^2 \geq 0 \Rightarrow \underbrace{(x+2)^2 + 4}_{h(x)} \geq 4$

range: $[4, +\infty)$

d) $i(x) = \frac{4x}{x+6}$

$i(x)$ is a one to one function and has an inverse.

find inverse: $y = \frac{4x}{x+6} \Rightarrow 4x = yx + 6y$

$\Rightarrow x = \frac{6y}{4-y}$

change $x \rightarrow y$
 $y \rightarrow x$

$i^{-1}(x) = \frac{6x}{4-x}$

domain of $i^{-1}(x)$ is the range of $i(x)$. range

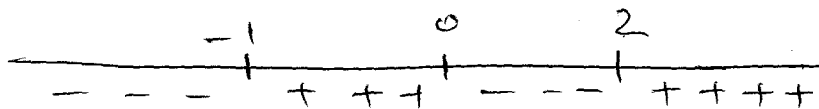
e) $j(x) = 3x^4 - 4x^3 - 12x^2$

$(-\infty, 4) \cup (4, +\infty)$

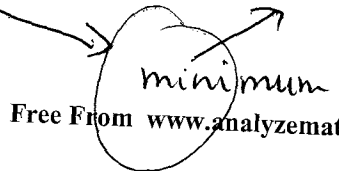
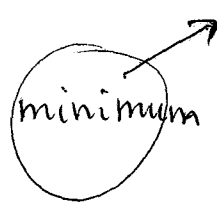
leading coefficient is positive, degree 4, graph of $j(x)$ rises up both sides. $j(x)$ has minimum value(s).

derivative: $j'(x) = 12x^3 - 12x^2 - 24x = 12x(x^3 - x^2 - 2) = 12x(x+1)(x-2)$

sign of $j'(x)$



$j(x)$



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2 minima

$j(-1) = -5$

$j(2) = -32$

absolute minimum value of $j(x) = -32$

range: $[-32, +\infty)$

3. Let $f(x) = \ln(x), x > 0$ and $g(x) = 16 - x^2$.

Find the domain and range of $f(g(x))$

Domain of g : $(-\infty, +\infty)$

Domain of: $f(g(x)) = f(16 - x^2)$
 $= \ln(16 - x^2)$

$$16 - x^2 > 0 \Rightarrow -4 < x < 4$$

Domain of $f(g(x))$: $-4 < x < 4$

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$(-4, 4)$.