Calculus Worksheet: Domain and Range (1)

1. Find the domain of the following functions

a) $f(x) = \dfrac{1}{x-2}$

$x - 2 \neq 0 \Rightarrow x \neq 2$

domain: $(-\infty, 2) \cup (2, +\infty)$

b) $g(x) = \dfrac{1}{x^4 + 2}$

$x^4 + 2 > 0 \Rightarrow$ domain: $(-\infty, +\infty)$.

c) $h(x) = \ln(x^2 - 9)$

$x^2 - 9 > 0$ domain: $(-\infty, -3) \cup (3, +\infty)$

d) $i(x) = \sqrt{2 - \sqrt{x+2}}$

Condition (1): $x + 2 \geq 0 \Rightarrow x \geq -2$

Condition (2): $2 - \sqrt{x+2} \geq 0$

$2 \geq \sqrt{x+2}$

$4 \geq x + 2 \Rightarrow x \leq 2$

domain: $[-2, 2]$.

e) $j(x)=\dfrac{1}{x^3-2}$

$x^3-2 \neq 0 \Rightarrow x \neq \sqrt[3]{2}$

Domain: $(-\infty, \sqrt[3]{2}) \cup (\sqrt[3]{2}, +\infty)$

f) $k(x)=\dfrac{1}{2-\sqrt{x^2-1}}$

Condition (1): $x^2-1 \geq 0 \Rightarrow x \in (-\infty, -1] \cup [1, +\infty)$

Condition (2): $2-\sqrt{x^2-1} \neq 0 \Rightarrow 4 \neq x^2-1$

$x \neq \pm\sqrt{5}$

Domain:

$(-\infty, -\sqrt{5}) \cup (-\sqrt{5}, -1) \cup (1, \sqrt{5}) \cup (\sqrt{5}, +\infty)$

g) $l(x)=\dfrac{\sqrt{x+8}}{(x-6)\sqrt{|x|-4}}$

Condition (1): $x+8 \geq 0 \quad x \geq -8$

Condition (2): $x-6 \neq 0 \quad x \neq 6.$

Condition (3): $|x|-4 > 0 \Rightarrow x \in (-\infty, -4) \cup (4, +\infty)$

Domain: $[-8, -4) \cup (4, 6) \cup (6, +\infty)$

2. Find the range of the following functions

a) $f(x)=|x+2|-5$

$|x+2| \geq 0 \Rightarrow \underbrace{|x+2|-5}_{f(x)} \geq -5$

range: $[-5, +\infty)$.

b) $g(x)=2+\sqrt{x+5}$

$\sqrt{x+5} \geq 0 \Rightarrow \underbrace{2+\sqrt{x+5}}_{g(x)} \geq 2$

range: $[2, +\infty)$.

c) $h(x) = x^2 + 4x + 8$

$= (x+2)^2 + 4$ complete square

$(x+2)^2 \geq 0 \Rightarrow \underbrace{(x+2)^2 + 4}_{h(x)} \geq 4$

range: $[4, +\infty)$

d) $i(x) = \dfrac{4x}{x+6}$

$i(x)$ is a one to one function and has an inverse.

find inverse: $y = \dfrac{4x}{x+6} \Rightarrow 4x = yx + 6y$

$\Rightarrow x = \dfrac{6y}{4-y}$

Change $x \to y$
$y \to x$

$i^{-1}(x) = \dfrac{6x}{4-x}$

domain of $i^{-1}(x)$ is the range of $i(x)$.

range
$(-\infty, 4) \cup (4, +\infty)$

e) $j(x) = 3x^4 - 4x^3 - 12x^2$

leading coefficient is positive, degree 4, graph of $j(x)$ rises up both sides. $j(x)$ has minimum value(s).

derivative: $j'(x) = 12x^3 - 12x^2 - 24x = 12x(x^3 - x^2 - 2)$
$= 12x(x+1)(x-2)$

		-1		0		2	
sign of $j'(x)$	$- - -$		$+ + +$		$- - -$		$+ + + +$
$j(x)$	$\searrow$	minimum	$\nearrow$		$\searrow$	minimum.	$\nearrow$

2 minima

$j(-1) = -5$ $j(2) = -32$

absolute minimum value of $j(x) = -32$

range: $[-32, +\infty)$.

3. Let $f(x)=\ln(x), x>0$ and $g(x)=16-x^2$.

Find the domain and range of $f(g(x))$

Domain of g: $(-\infty, +\infty)$

Domain of: $f(g(x)) = f(16-x^2)$
$= \ln(16-x^2)$

$16-x^2>0 \Rightarrow -4<x<4$

Domain of $f(g(x))$: $\underline{-4<x<4}$

$(-4, 4)$.