Calculus Worksheet: Domain and Range Of Inverse Trigonometric Functions(1)

Review:

1. $y=\sin^{-1}(x)$ domain: $[-1,1]$ range: $[-\frac{\pi}{2},\frac{\pi}{2}]$
2. $y=\cos^{-1}(x)$ domain: $[-1,1]$ range: $[0,\pi]$
3. $y=\tan^{-1}(x)$ domain: $(-\infty,\infty)$ range: $(-\frac{\pi}{2},\frac{\pi}{2})$

Questions

1. Find the domain of the following functions

a) $f(x)=2\sin^{-1}(2x-1)$

Domain: $-1\le 2x-1\le 1 \Rightarrow 0\le x\le 1$ $[0,1]$

Range: $-\frac{\pi}{2}\le \sin^{-1}(2x-1)\le \frac{\pi}{2} \Rightarrow -\pi\le 2\sin^{-1}(2x-1)\le\pi$

$[-\pi,\pi]$.

b) $g(x)=3\tan^{-1}(x)$

Domain: $(-\infty,+\infty)$

Range: $(-\frac{3\pi}{2},\frac{3\pi}{2})$

c) $h(x)=\pi-2\cos^{-1}(-x+1)$

Domain: $-1\le -x+1\le 1 \Rightarrow -2\le -x\le 0$

$2\ge x\ge 0$

$[0,2]$.

Range: $0\le \cos^{-1}(-x+1)\le\pi$

$0\ge -2\cos^{-1}(-x+1)\ge -2\pi$

$\pi\ge \pi-2\cos^{-1}(-x+1)\ge -\pi$ $[-\pi,\pi]$

d) $i(x) = \sin(\tan^{-1}(x))$

Domain: $(-\infty, +\infty)$

Range: $-\frac{\pi}{2} < \tan^{-1}(x) < \frac{\pi}{2}$

$\Rightarrow \sin(-\pi/2) < \sin(\tan^{-1}(x)) < \sin\frac{\pi}{2}$

$-1 \qquad\qquad 1$

e) $j(x) = \cos(\sin^{-1}(x))$ Range: $(-1, 1)$

Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le \sin^{-1}(x) \le \frac{\pi}{2}$

$0 \le \cos(\sin^{-1}(x)) \le 1$ $\quad [0, 1]$

f) $k(x) = \tan(\cos^{-1}(x))$

Domain:

$x \in [-1, 1]$ but $x \neq 0$ why?

$\cos^{-1}(0) = \frac{\pi}{2}$ and $\tan(\frac{\pi}{2})$ is undefined.

hence domain: $[-1, 0) \cup (0, +1]$.

Range: $\tan(\pi/2)$ is undefined

however there is a vertical asymptote

at $x = 0$ since $\cos^{-1}(0) = \pi/2$

hence range: $(-\infty + \infty)$