

Calculus Worksheet: Even and Odd Functions (1)

1. f is an even function and g is an odd function.

Show that

- a) $(f \cdot g)(x)$ is an odd function.

$$f \text{ even} \Rightarrow f(-x) = f(x) \quad ; \quad g \text{ odd} \Rightarrow g(-x) = -g(x)$$

$$\text{let } h(x) = (f \cdot g)(x) = f(x) \cdot g(x)$$

$$h(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -f(x) \cdot g(x)$$

$$h(-x) = -h(x) \quad h(x) = (f \cdot g)(x) \text{ is odd.}$$

- b) $(f/g)(x)$ is an odd function.

$$\text{let } k(x) = (f/g)(x) = \frac{f(x)}{g(x)}$$

$$k(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -k(x)$$

$$\Rightarrow k(x) = (f/g)(x) \text{ is odd.}$$

2. Show that $f(|x|)$ is an even function.

$$\text{let } h(x) = f(|x|)$$

$$h(-x) = f(|-x|) = f(|x|) \quad \text{since } |-x| = |x|$$

$$\Rightarrow h(x) = h(-x) \Rightarrow h(x) = f(|x|) \text{ is even}$$

3. Show that if f and g are even functions, then $(f + g)$ is also even.

$$\text{let } h(x) = (f + g)(x) = f(x) + g(x)$$

$$h(-x) = f(-x) + g(-x) = f(x) + g(x) \quad \leftarrow \text{f and g even.}$$

$$\Rightarrow h(x) = h(-x) \Rightarrow h(x) = (f + g)(x) \text{ is even.}$$

4. Show that if f and g are odd functions, then $(f + g)$ is also odd.

$$\text{let } k(x) = (f + g)(x) = f(x) + g(x) \quad \leftarrow \text{f and g odd.}$$

$$k(-x) = f(-x) + g(-x) = -f(x) - g(x)$$

$$k(-x) = -(f(x) + g(x)) = -k(x)$$

$$\Rightarrow k(x) = (f + g)(x) \text{ is odd.}$$