Calculus Worksheet: Integration of Functions (1)

Use integration rules to evaluate the following integrals

1. $\int \frac{2}{(x-3)(x+2)}dx$

Use partial fractions decomposition on

$$\frac{2}{(x-3)(x+2)} = \frac{2}{5(x-3)} - \frac{2}{5(x+2)}$$

Hence

$$\int \frac{2}{(x-3)(x+2)}dx = \int \frac{2}{5} \cdot \frac{1}{x-3}dx - \int \frac{2}{5} \cdot \frac{1}{x+2}dx$$

$$= \frac{2}{5} \ln(x-3) - \frac{2}{5} \ln(x+1)$$

$$= \frac{2}{5} \ln\left(\frac{x-3}{x+2}\right) + C$$

2. $\int \frac{2}{(x-3)\ln|x-3|}dx$

let $u = \ln|x-3|$.

$$\frac{du}{dx} = \frac{1}{x-3} \Rightarrow du = \frac{dx}{x-3}$$

$$\Rightarrow \int \frac{2}{(x-3)\ln|x-3|}dx = 2\int \frac{du}{u} = 2 \ln u.$$

$$= 2\ln(\ln|x-3|) + C$$

3. $\int x \sin x dx$

let $f(x) = x$ and $g(x) = \sin x$

Apply integration by parts

$$\int x \cdot \sin(x)\, dx = x \cdot (-\cos x) - \int -\cos x \cdot 1 \cdot dx$$

$$= -x \cos x + \sin x + C$$

4. $\int \tan(2x)dx$

let $u = 2x$ $\qquad \dfrac{du}{dx} = 2.$

$$dx = \frac{du}{2}.$$

$$\Rightarrow \int \tan(2x)\, dx = \int \tan u \cdot \frac{du}{2}.$$

$$= \frac{1}{2}\int \frac{\sin u}{\cos u} \cdot du$$

let $V = \cos u \Rightarrow \dfrac{dV}{du} = -\sin u.$

$$= \frac{1}{2}\int - \frac{dV}{V} = -\frac{1}{2} \ln V$$

$$= -\frac{1}{2} \ln \cos(2x) + C$$

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5. $\displaystyle\int \frac{x^2+x}{(x^2+2)(x+3)}\,dx$

fractions decomposition

$$\frac{x^2+x}{(x^2+2)(x+3)} = \frac{5x}{11(x^2+2)} - \frac{4}{11(x^2+2)} + \frac{6}{11(x+3)}$$

$$\int \frac{5x}{11(x^2+2)}\,dx = \frac{5\ln(x^2+2)}{22} + C$$

$$\int \frac{4}{11(x^2+2)}\,dx = \frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{11}$$

$$\int \frac{6}{11(x+3)}\,dx = \frac{6}{11}\ln(x+3)$$

Hence $\displaystyle\int \frac{x^2+x}{(x^2+2)(x+3)}\,dx$

$$= \frac{5\ln(x^2+2)}{22} + \frac{6}{11}\ln(x+3) - \frac{2\sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right)}{11} + C$$

6. $\displaystyle\int e^{(\ln x + x^2)}\,dx$

Note that

$$e^{\ln x + x^2} = e^{\ln x} \cdot e^{x^2} = x \cdot e^{x^2}$$

$$= \int x \cdot e^{x^2}\,dx$$

let $u = x^2$, $\frac{du}{dx} = 2x$

$$= \int \frac{1}{2} \cdot e^u \, du = \frac{1}{2} e^u = \frac{1}{2} e^{x^2} + C$$

7. $\int \frac{2}{x^2-2x+3}dx$

Complete the square / denominator.

$x^2 - 2x + 3 = (x-1)^2 + 2$

$= \int \frac{2}{(x-1)^2+2} dx$, let $u = x-1$.
$du = dx$

$= \int \frac{2}{u^2+2} du = \int \frac{1}{\frac{u^2}{2}+1} du$ let $v = \frac{u}{\sqrt{2}}$
$dv = \frac{du}{\sqrt{2}}$

$= \int \frac{1}{v^2+1} \sqrt{2} \cdot dv$

$= \sqrt{2} \arctan(v)$

$= \sqrt{2} \arctan\left(\frac{x-1}{\sqrt{2}}\right) + C$

8. $\int \frac{2}{\sqrt{1-2x-x^2}}dx$

Complete Square

$1 - 2x - x^2 = -(x+1)^2 + 2$

$= \int \frac{2}{\sqrt{2-(x+1)^2}} dx$

let $u = x+1 \Rightarrow du = dx$

$= \int \frac{2}{\sqrt{2-u^2}} du$

$= \int \frac{\sqrt{2}}{\sqrt{1-\left(\frac{u}{\sqrt{2}}\right)^2}} du$, let $v = \frac{u}{\sqrt{2}}$

$= \sqrt{2} \cdot \sqrt{2} \cdot \arcsin\left(\frac{x+1}{\sqrt{2}}\right) + C$