Calculus Worksheet: Limits of Functions (1)

Find the following limits

1. $\lim\limits_{x\to+\infty} \dfrac{x^2+2x-3}{-5x^2}$

factor x^2 and simplify

$$= \lim_{x\to+\infty} \frac{x^2\left(1+\frac{2}{x}-\frac{3}{x^2}\right)}{x^2(-5)}$$

as $x \to +\infty$, $\frac{2}{x}$ and $\frac{3}{x^2}$ approaches zero.

$$= \lim_{x\to+\infty} \left(\frac{1}{-5}\right) = -\frac{1}{5}$$

2. $\lim\limits_{x\to-\infty} \dfrac{x^4+2x^2-3x}{4x^3}$

factor x^3 and simplify.

$$= \lim_{x\to-\infty} \frac{x^3\left(x+\frac{2}{x^2}-\frac{3}{x^3}\right)}{x^3(4)}$$

as $x \to -\infty$, $\frac{2}{x^2}$ and $\frac{3}{x^3}$ approaches 0.

$$= \lim_{x\to-\infty} \left(\frac{x}{4}\right) = -\infty .$$

3. $\lim_{x\to-\infty}\frac{x-1}{|x+2|}$

as $x \to 0 - \infty$, $x+2$ also approaches $-\infty$.

$|x+2| \to 0 - (x+2)$.

$$= \lim_{x\to 0-\infty} \frac{(x-1)}{-(x+2)} = \lim \frac{x\left(1-\frac{1}{x}\right)}{x\left(-1-\frac{2}{x}\right)}.$$

$$= \lim_{x\to 0-\infty} \left(\frac{1}{-1}\right) = \underline{-1}.$$

4. $\lim_{x\to 2^-}\frac{x^2+x-6}{|x-2|}$

As $x \to 2^-$ $\quad x-2 < 0$ and $|x-2| = -(x-2)$

$$= \lim_{x\to 2^-} \frac{x^2+x-6}{-(x-2)} = \lim_{x\to 2^-} \frac{(x-2)(x+3)}{-(x-2)}$$

$$= \lim_{x\to 2^-} \frac{x+3}{-1} = \underline{-5}$$