Calculus Worksheet: Limits of Functions (2)

Find the following limits

1. $$\lim_{x\to 0}\frac{x^2-16}{x^2+2x-24} = \frac{-16}{-24} = 2/3$$

0 0 0

2. $$\lim_{x\to -1}\frac{x^2-4x-5}{|x+1|}$$

We need to find the above limit as

$x \to -1^-$ and $x \to -1^+$

(A) As $x \to -1^+$ right $x+1>0 \Rightarrow |x+1| = x+1$

$$= \lim_{x\to -1^+}\frac{x^2-4x-5}{x+1} = \lim_{x\to -1^+}\frac{\cancel{(x+1)}(x-5)}{\cancel{(x+1)}} = \underline{-6}$$

(B) As $x \to -1^-$ left $x+1<0 \Rightarrow |x+1| = -(x+1)$

$$= \lim_{x\to -1^-}\frac{x^2-4x-5}{-(x-1)} = \lim_{x\to -1^-}\frac{(x+1)(x-5)}{-(x+1)} = \underline{6}$$

The limits from the left and from the right are different hence the limit <u>does not exist</u>.

3. $\lim\limits_{x \to -3^+} \sqrt[3]{x+3}\ln(x+3) = 0 \cdot \infty$ indeterminate form.

$$= \lim_{x \to -3^+} \frac{\ln(x+3)}{(x+3)^{-1/3}} = \frac{\infty}{\infty}$$

Use the l'hopital rule

$$= \lim_{x \to -3^+} \frac{\frac{1}{x+3}}{-\frac{1}{3}(x+3)^{-4/3}} = -3 \frac{(x+3)^{-1}}{(x+3)^{-4/3}}$$

$$= \lim_{x \to -3^+} -3(x+3)^{1/3} = 0.$$

4. $\lim\limits_{x \to 4} \frac{\sqrt{x}-2}{x-4} = \frac{0}{0}$ indeterminate form.

multiply numerator and denominator by $\sqrt{x}+2$.

$$= \lim_{x \to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)} = \lim_{x \to 4} \frac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)}$$

$$= \lim_{x \to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}.$$