

Calculus Worksheet: Limits of Functions (4)

Find the following limits

$$1. \lim_{x \rightarrow +\infty} \frac{-7x}{\sqrt{4x^2 + 3}} = \frac{-7x}{2|x| \sqrt{1 + \frac{3}{4x^2}}}$$

as  $x \rightarrow +\infty$ ,  $\frac{3}{4x^2} \rightarrow 0$  and  $|x| \rightarrow x$

$$= \lim_{x \rightarrow +\infty} \frac{-7x}{2x \sqrt{1 + \frac{3}{4x^2}}} = -7/2.$$

$$2. \lim_{x \rightarrow \infty} \sqrt{x^2 + x + 2} - x = \lim_{x \rightarrow \infty} |x| \sqrt{1 + \frac{1}{x} + \frac{2}{x^2}} - x$$

=  $\infty - \infty$  indeterminate form

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + x + 2} - x)(\sqrt{x^2 + x + 2} + x)}{\sqrt{x^2 + x + 2} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + x + 2 - x^2}{\sqrt{x^2 + x + 2} + x} = \lim_{x \rightarrow \infty} \frac{x + 2}{\sqrt{x^2 + x + 2} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x(1 + \frac{2}{x})}{x[\sqrt{1 + \frac{1}{x} + \frac{2}{x^2}} + 1]} = \frac{1}{2}$$