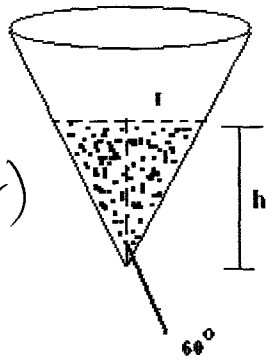
Calculus Worksheet: Rate of Change (1)

Water is poured at the rate of *0.01* liter/second into the conical container shown below. Assume the container is empty at the start of the experiment (t = 0), find the rate of change of h the height of the water in the container at t = 3 seconds.

Note: 1 liter = $1\,(dm)^3$

all length will be in dm. (decimeter)

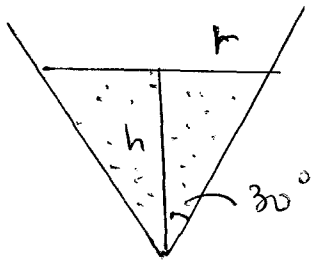


Water poured at rate of 0.01 L/s $\Rightarrow$ Volume V of water is given by.

$V = 0.01 \cdot t$ V in liters, t in seconds

Also $V = \frac{1}{3}\pi r^2 h$.

$\tan 30° = r/h \Rightarrow r = h \tan 30°$



$\Rightarrow V = \frac{1}{3}\pi (h \tan 30°)^2 \cdot h = 0.01\,t$

$\Rightarrow h^3 \frac{\pi \tan^2 30°}{3} = 0.01\,t$

Solve for h.

$h = \sqrt[3]{\frac{0.03\,t}{\pi \tan^2(30°)}} = \left(\frac{0.03}{\pi \tan^2 30°}\right)^{1/3} t^{1/3}$

$\frac{dh}{dt} = \frac{1}{3}\left(\frac{0.03}{\pi \tan^2 30°}\right)^{1/3} \cdot t^{-2/3}$

at $t = 3$ sec $\frac{dh}{dt} = 0.05$ dm/sec.