

Calculus Worksheet: Rate of Change (2)

A block of ice start melting at  $t = 0$  where  $t$  is the time. The rate at which its mass  $M$  decreases is proportional to the cube root of the mass. If the initial (at  $t = 0$ ) mass  $M$  is equal to 5 Kilograms and it is equal to 3 Kilograms at  $t = 1$  hour, find  $t$  so that  $M = 0$ .

$$\frac{dM}{dt} = k \sqrt[3]{M} \quad : \text{rate proportional to cube root of } M. \\ k = \text{constant.}$$

$$\Rightarrow M^{-\frac{1}{3}} \cdot \frac{dM}{dt} = k$$

integrate both sides w.r. to  $t$ :

$$\int M^{-\frac{1}{3}} \frac{dM}{dt} \cdot dt = \int k dt$$

↓

$$\frac{2}{2} M^{2/3} = kt + c' \quad , c' \text{ constant of integration}$$

$$M = \sqrt{\left(\frac{2}{3} kt + c\right)^3} \quad , c \text{ constant} = \frac{2}{3} c'$$

$$M(0) = 5 = \sqrt{c^3} \quad (\text{given})$$

$$\Rightarrow c = \sqrt[3]{25}$$

Free calculus worksheets from [www.analyzemath.com](http://www.analyzemath.com)

$$\Rightarrow M(t) = \sqrt{\left(\frac{2}{3} kt + \sqrt[3]{25}\right)^3}$$

$$M(1) = \sqrt{\left(\frac{2}{3} k(1) + \sqrt[3]{25}\right)^3} = 3 \quad \text{given.}$$

$$\text{solve for } k : k = \frac{3}{2} \left[ \sqrt[3]{9} - \sqrt[3]{25} \right].$$

$$M(t) = \sqrt{\left[\left(\sqrt[3]{9} - \sqrt[3]{25}\right)t + \sqrt[3]{25}\right]^3}$$

set  $M(t) = 0$  and solve for  $t$  to obtain

$$t = \frac{-\sqrt[3]{25}}{\sqrt[3]{9} - \sqrt[3]{25}} \approx 3.5 \text{ hours}$$