Calculus Worksheet: Rate of Change (2)

A block of ice start melting at t = 0 where t is the time. The rate at which its mass M decreases is proportional to the cube root of the mass. If the initial (at t = 0) mass M is equal to 5 Kilograms and it is equal to 3 Kilograms at t = 1 hour, find t so that M = 0.

$$\frac{dM}{dt} = k\sqrt[3]{M}$$: rate proportional to cube root of M. k = constant.

$$\Rightarrow M^{-\frac{1}{3}} \cdot \frac{dM}{dt} = k$$

integrate both sides w.r.t t.:

$$\int M^{-\frac{1}{3}} \frac{dM}{dt} \cdot dt = \int k\, dt$$

$$\Downarrow$$

$$\frac{3}{2} M^{2/3} = kt + c'$$, c' constant of integration

$$M = \sqrt{\left(\frac{2}{3}kt + c\right)^3}$$, c constant $= \frac{2}{3}c'$.

$$M(0) = 5 = \sqrt{c^3}$$ (given)

$$\Rightarrow c = \sqrt[3]{25}$$

$$\Rightarrow M(t) = \sqrt{\left(\frac{2}{3}kt + \sqrt[3]{25}\right)^3}$$

$$M(1) = \sqrt{\left(\frac{2}{3}k(1) + \sqrt[3]{25}\right)^3} = 3$$ given.

solve for k : $k = \frac{3}{2}\left[\sqrt[3]{9} - \sqrt[3]{25}\right]$.

$$M(t) = \sqrt{\left[\left(\sqrt[3]{9} - \sqrt[3]{25}\right)t + \sqrt[3]{25}\right]^3}$$

set $M(t) = 0$ and solve for t to obtain $t = \dfrac{-\sqrt[3]{25}}{\sqrt[3]{9} - \sqrt[3]{25}} \approx 3.5$ hours