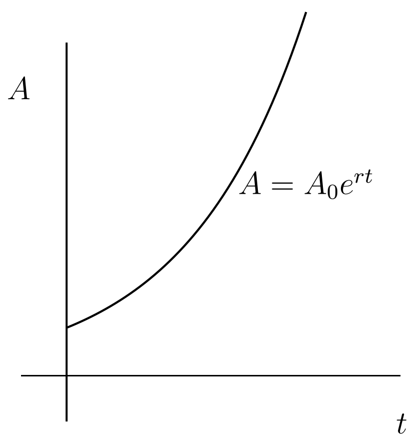
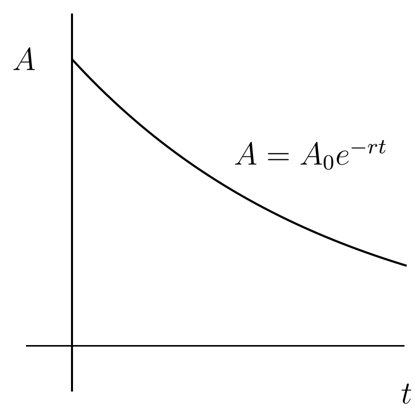
College Algebra Worksheet (2)

Exponential Growth and Decay Problems

If a certain quantity A is growing continuously at rate r, then A may be written as a function of time as follows $A = A_0 e^{rt}$. If A is decaying continuously at rate r, then A may be written as follows $A = A_0 e^{-rt}$. The positive constant r is called either the growth rate (for exponential growth) or the decay constant (for exponential decay). A_0 is the initial amount of A (i.e. the amount A when $t = 0$ or $A_0 = A(0)$).





The half life of a decaying material is the time T it takes a certain amount of this material modeled by $A = A_0 e^{-rt}$ to become $\dfrac{A_0}{2}$. T may be found as follows

$A_0 e^{-rT} = \dfrac{A_0}{2}$

divide both sides by A_0 and solve for T

$e^{-rT} = \dfrac{1}{2}$

$-rT = ln\dfrac{1}{2}$

$-rT = -ln2$

$$T = \frac{ln2}{r}$$

Applications of Growth and Decay Models

Many natural phenomena and man made physical systems are modeled by the same growth and decay models seen above.

1. **exponential growth**
   - population growth
   - compound interest
   - charge across capacitor in an electrical circuit after power is switched on.
2. **exponential decay examples**
   - radioactive decay
   - concentration of medicine in the body
   - charge across capacitor in an electrical circuit after power is switched off.
   - atmospheric pressure as a function of altitude

Example 1
At t = 0 there are 50 grams of a radioactive isotope. The isotope has a half-life of 16 minutes. Use the exponential decay model to write the amount A as a function of time t.

Solution

We first use the half life formula $T = \dfrac{ln2}{r}$ to calculate r.

$$r = \frac{ln2}{T} = 0.04332\ minutes^{-1}$$

Hence $A = A_0 e^{-rt} = 50e^{-0.04332t}$, where A is in grams and t in minutes.

Problems

1. In the year 2000 the population of a large city was 25 million and increasing continuously at the rate of 2.5% per year.

a What was the population of this city in the year 2005?

b Assuming that the population will continue increasing continuously at the same rate, when will the population reach 50 million?

2. The population of a certain country is growing exponentially according to the model $P = P_0 e^{rt}$ where P is the population in millions and t is the number of years after 1980. In 1988 the population was 35 million, and in 1995 the population was 48 million. What is the growth rate r?

3. At time $t = 0$ you have 100 grams of radioactive Chromium-48. Ten hours later you have 74 grams left. What is the half life of Chromium-48?

4. The radioactive isotope Cobalt-60 (Co-60) has a half-life of 5.24 years. You have an initial amount of 80 grams.

a Write the amount as a function of time (in years).

b How much Co-60 is left after 2 years?

c How much time has passed when 40% of the initial amount is left?

5. The atmospheric pressure at an altitude x above sea level decreases according to the model $P = P_0 e^{-kx}$, where $P_0 = 14.7\, psi$ is the pressure at sea level and $k = 3.9810^{-5}\ ft^{-1}$.

a What is the atmospheric pressure on top of Mount Everest (29000 ft)?.

b Solve for x in terms of P, P_0 and k.

c At which altitude is the atmospheric pressure 11.5 psi?

6. A population of bacteria is growing exponentially. It takes the population 15 minutes to double. How long will it take for the population to triple (3)?.