

Math Worksheet: Inverse Functions (2)

1. Find the inverse of each function and check your answer.

A. $f(x) = -2 \ln(2x-3) + 2$

$$y = -2 \ln(2x-3) + 2$$

interchange x and y and solve for y :

$$x = -2 \ln(2y-3) + 2 \Rightarrow \ln(2y-3) = \frac{2-x}{2}$$

$$\Rightarrow 2y-3 = e^{\frac{2-x}{2}} \Rightarrow y = f^{-1}(x) = \frac{1}{2} e^{\frac{2-x}{2}} + \frac{3}{2}$$

check:

$$f(f^{-1}(x)) = -2 \ln\left(2\left(\frac{1}{2} e^{\frac{2-x}{2}} + \frac{3}{2}\right) - 3\right) + 2$$

$$= -2 \ln\left(e^{\frac{2-x}{2}}\right) + 2 = -2\left(\frac{2-x}{2}\right) + 2$$

$$= x$$

$$f^{-1}(f(x)) = \frac{1}{2} e^{\frac{1}{2}(x+2 \ln(2x-3)+2)} + \frac{3}{2} = \frac{1}{2} e^{\ln(2x-3)} + \frac{3}{2}$$

$$= \frac{1}{2} (2x-3) + \frac{3}{2} = x$$

B. $h(x) = 2 + e^{(3x-2)+4}$

$$y = 2 + e^{3x+2}$$

interchange x and y and solve: $x = 2 + e^{3y+2} \Rightarrow e^{3y+2} = x-2$

$$3y+2 = \ln(x-2) \Rightarrow y = h^{-1}(x) = \frac{1}{3} (\ln(x-2) - \frac{2}{3})$$

check:

$$h(h^{-1}(x)) = 2 + e^{(3(\frac{1}{3} \ln(x-2) - \frac{2}{3}) - 2) + 4}$$

$$= 2 + e^{\ln(x-2)} = 2 + x - 2 = x$$

$$h^{-1}(h(x)) = \frac{1}{3} \ln(2 + e^{3x+2} - 2) - \frac{2}{3} = \frac{1}{3} (\ln e^{3x+2} - \frac{2}{3}) = x$$