

Math Worksheet: Graphs of Rational Functions(2)

Given the function

$$f(x) = \frac{x^3 - 1}{x^2 - 4}$$

1. Find the domain
- f
- .

Domain: $x^2 - 4 = 0 \Rightarrow x = \pm 2$

Domain: all real numbers except ± 2
or $(-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$.

2. Find the vertical asymptotes and oblique (slant) of the graph of
- f
- .

V. asymptotes: $x^2 - 4 = 0 \Rightarrow x = \pm 2$

oblique asymptote: $\frac{x^3 - 1}{x^2 - 4} = x + \frac{4x - 1}{x^2 - 4}$

$\rightarrow y = x \leftarrow$

3. Find the y-intercept and x intercept, if any, of the graph of
- f
- .

y-intercept: $f(0) = \frac{1}{4}$. $(0, \frac{1}{4})$

x-intercept: $x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$

one solution only: $x = 1$ $(1, 0)$

4. For what values of
- x
- is
- $f(x)$
- positive?

Note that: $x^3 - 1 = (x - 1)(x^2 + x + 1)$
and that $x^2 + x + 1 > 0$ for all x values.

hence

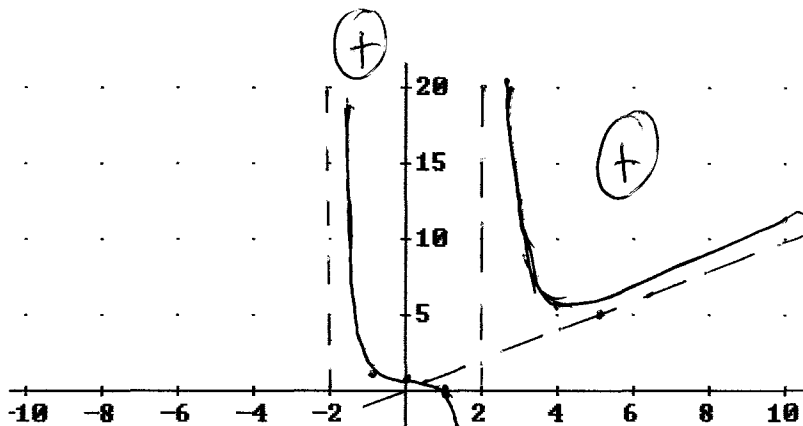
	-2	1	2
$x^2 + x + 1$	+	+	+
$x - 1$	-	-	+
$x - 2$	-	-	+
$x + 2$	-	+	+
$f(x)$	-	+	+

Table of Sign

$$f(x) = \frac{(x-1)(x^2+x+1)}{(x-2)(x+2)}$$

5. Sketch the graph of f .

Use oblique and vertical asymptote + table of sign + Extra points.



oblique asymptote
 $y = x$.

Vertical asymptotes
 $x = 2$
 $x = -2$

more points

x	$f(x)$
-4	-5.4
-1	0.6
4	5.3