Calculus Worksheet: Derivatives of Stepwise Functions (1)

If

\[ f(x) = \begin{cases} 
  x^3, & x \leq 1 \\
  3x - 2, & x > 1 
\end{cases} \]

does \( f'(1) \) exist? If yes find its value.

\[ \lim_{x \to 1^-} f(x) = 1 \]

\[ \lim_{x \to 1^+} f(x) = 1 \]

also \( f(1) = 1 \).

\( f(x) \) is defined and continuous for all \( x \) including \( x = 1 \).

Also:

\[ f'(x) = 3x^2, \quad x < 1 \]

\[ f'(x) = 3, \quad x > 1 \]

\[ \lim_{x \to 1^-} f'(x) = 3 \]

\[ \lim_{x \to 1^+} f'(x) = 3 \]

hence

\[ \lim_{x \to 1} f'(x) = 3 \]

\[ f'(1) = 3 \]