Math Worksheet: Geometry Problem (3)

1. ABC is a right triangle and D is a point on BC at a distance of 1/2 unit from point B. Angle BAD is 30 degrees and ADE is a right angle. The distance CE is equal to 1 unit. Find x the distance from A to E.

We First draw the perpendicular EE' to BC. Hence triangles CEE' and CAB are similar which gives

$$\frac{1}{1+x} = \frac{EE'}{AB} \quad (1)$$

We now use sine and cosine ratios to find AB and AD as follows

$$\sin 30° = \frac{1}{2} = \frac{1/2}{AD} \implies AD = 1$$

$$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AB}{1} \implies AB = \sqrt{3}/2 \text{ and } (1) \implies EE' = \frac{\sqrt{3}/2}{1+x}$$

$$\angle BAC = 30° \implies \angle BDA = 60° \implies \angle E'DE = 30°$$

$$\implies \angle E'ED = 60° .$$

Right Triangles BAD and $E'DE$ have equal angles and are therefore similar

hence $\dfrac{EE'}{1/2} = \dfrac{DE}{1} \implies DE = 2EE' = \dfrac{\sqrt{3}}{1+x}$

Pythagora's theorem applied to $\triangle ADE$ gives

$$AD^2 + DE^2 = x^2$$

$$1^2 + \left(\frac{\sqrt{3}}{1+x}\right)^2 = x^2 \implies x^4 + 2x^3 - 2x - 4 = 0$$

$$\implies x^3(x+2) - 2(x+2) = 0$$

$$(x+2)(x^3 - 2) = 0$$

$$\implies x = \sqrt[3]{2}$$