Math Worksheet: Graphs of Rational Functions(2)

Given the function

$$f(x)=\frac{x^3-1}{x^2-4}$$

1. **Find the domain f.**

Domain : $x^2-4=0 \Rightarrow x=\pm 2$

Domain : all real numbers except ± 2
or $(-\infty,-2)\cup(-2,2)\cup(2,+\infty)$.

2. **Find the vertical asymptotes and oblique (slant) of the graph of f.**

V. asymptotes : $x^2-4=0 \Rightarrow x=\pm 2$

oblique asymptote : $\frac{x^3-1}{x^2-4} = x + \frac{4x-1}{x^2-4}$

$y=x$

3. **Find the y-intercept and x intercept, if any, of the graph of f.**

y-intercept : $f(0)=1/4$. $(0,1/4)$.

x-intercept : $x^3-1=(x-1)(x^2+x+1)=0$

one solution only : $x=1$ $(1,0)$

4. **For what values of is $f(x)$ positive?**

Note that : $x^3-1=(x-1)(x^2+x+1)$
and that $x^2+x+1>0$ for all x values.

table of sign

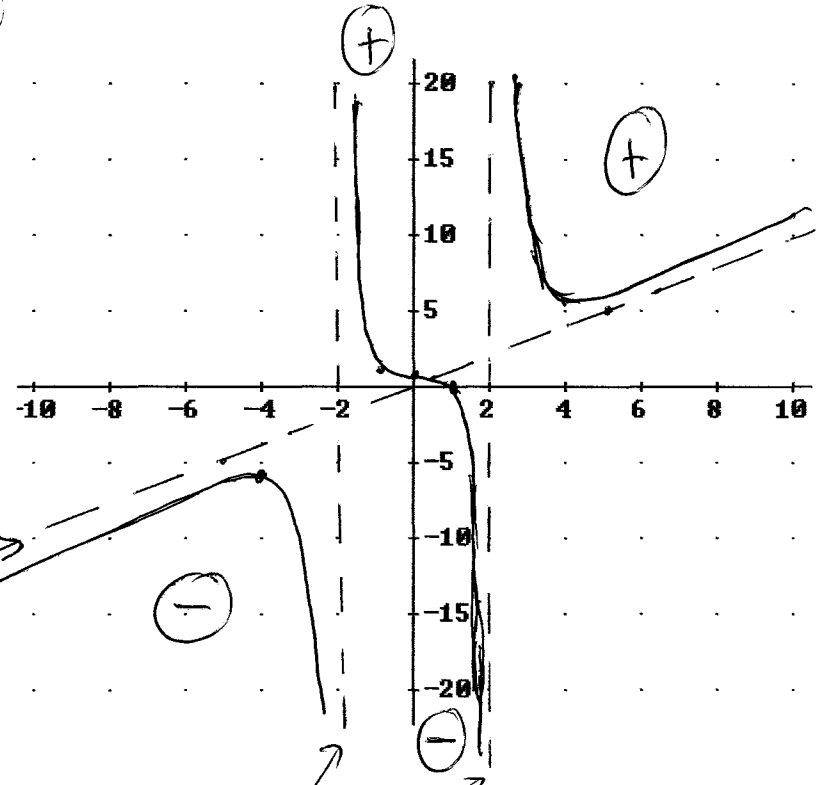
hence

$$f(x)=\frac{(x-1)(x^2+x+1)}{(x-2)(x+2)}$$

		-2		1		2	
x^2+x+1	+		+		+		+
$x-1$	−		−		+		+
$x-2$	−		−		−		+
$x+2$	−		+		+		+
$f(x)$	−		+		−		+

5. Sketch the graph of f .

Use oblique and vertical asymptote + table of sign + Extra points.



oblique asymptote $y = x$.

Vertical asymptotes $x = 2$, $x = -2$

more points

x	$f(x)$
-4	-5.4
-1	0.6
4	5.3