Simplify the following expression

\[
\frac{2x}{x^2 - 6x + 9} - \frac{1}{x + 1} - \frac{8}{x^2 - 2x - 3}
\]

Solution

factor the denominators

\[x^2 - 6x + 9 = (x - 3)^2\]

\[x^2 - 2x - 3 = (x + 1)(x - 3)\]

The original expression may now be written as

\[
\frac{2x}{x^2 - 6x + 9} - \frac{1}{x + 1} - \frac{8}{x^2 - 2x - 3} = \frac{2x}{(x-3)^2} - \frac{1}{x+1} - \frac{8}{(x+1)(x-3)}
\]

Common denominator

\[
= \frac{\frac{2x}{(x-3)^2} - \frac{1}{x+1} - \frac{8}{(x+1)(x-3)}}
= \frac{2x(x+1) - (x-3)^2 - 8(x+1)}{(x+1)(x-3)^2}
= \frac{x^2 + 15}{(x+1)(x-3)^2}
\]