Trigonometry Worksheet: Addition Formula (1)

1. Use the fact that $\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$ to find $\sin(\frac{\pi}{12})$.

$$\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{\pi}{3}\cdot\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{3}\cdot\sqrt{2}-\sqrt{2}}{4}$$

$$=\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}.$$

2. x is a third quarter angle such that $\sin(x)=\frac{-1}{5}$ and y is a second quarter angle such that $\cos(y)=\frac{-1}{3}$. Find $\sin(x+y)$.

$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y$

$\cos(x)$ is given by $\cos(x)=\pm\sqrt{1-\sin^2(x)}=-\frac{\sqrt{24}}{5}$

$x\in$ Q III.

$\sin y$ is given by $\sin y=\pm\sqrt{1-\cos^2 y}=\frac{\sqrt{8}}{3}$

$y\in$ Q II.

$$\sin(x+y)=\left(-\frac{1}{5}\right)\left(-\frac{1}{3}\right)+\left(-\frac{\sqrt{24}}{5}\right)\left(\frac{\sqrt{8}}{3}\right)$$

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$$=\frac{1-8\sqrt{3}}{15}.$$