

Trigonometry Worksheet: Half Angle Formula (1)

1. Use the half angle formulas to find the exact values of $\sin(15^\circ)$, $\cos(\frac{\pi}{8})$ and $\tan(\frac{3\pi}{8})$.

A) Half angle formula for sine is given

by
$$\sin\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

let $\alpha = 30^\circ$, we have

$$\sin(15^\circ) = \pm \sqrt{\frac{1 - \cos 30^\circ}{2}}$$

15° is in quadrant (I) $\Rightarrow \sin 15^\circ > 0$

$$\Rightarrow \sin(15^\circ) = + \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

B) Half angle formula for cosine

$$\cos\left(\frac{\alpha}{2}\right) = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\text{let } \alpha = \frac{\pi}{4}$$

$$\cos\left(\frac{\pi}{8}\right) = \pm \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}}$$

$$\frac{\pi}{8} \text{ is in quadrant (I)} \Rightarrow \cos\left(\frac{\pi}{8}\right) > 0$$

$$\Rightarrow \cos\left(\frac{\pi}{8}\right) = + \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

Half angle formula for tangent

$$\tan\left(\frac{\alpha}{2}\right) = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

$$\text{let } \alpha = \frac{3\pi}{4}$$

$$\Rightarrow \tan\left(\frac{3\pi}{8}\right) = \frac{1 - \cos\left(\frac{3\pi}{4}\right)}{\sin\left(\frac{3\pi}{4}\right)} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{\frac{\sqrt{2}}{2}}$$

$$= \frac{2 + \sqrt{2}}{\sqrt{2}} = \underline{1 + \sqrt{2}}$$