

Trigonometry Worksheet: Verify Identities (3)

Verify the identities

1.
$$\csc x + \cot x = \frac{\sin x}{1 - \cos x}$$

start with the right side and multiply numerator and denominator by $(1 + \cos(x))$:

$$\begin{aligned} \frac{\sin(x)}{1 - \cos(x)} &= \frac{\sin(x)(1 + \cos(x))}{(1 - \cos(x))(1 + \cos(x))} = \frac{\sin x + \sin x \cos x}{1 - \cos^2(x)} \\ &= \frac{\sin x}{\sin^2(x)} + \frac{\sin x \cos x}{\sin^2 x} = \frac{1}{\sin x} + \frac{\cos x}{\sin x} = \underline{\csc x + \cot(x)} \end{aligned}$$

2.
$$\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

start with left side and divide numerator and denominator by $\cos(x) \cdot \cos y$

$$\begin{aligned} \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} &= \frac{\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}}{\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}} \\ &= \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x}{\cos x} \cdot \frac{\sin y}{\cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y} \end{aligned}$$